NIMCET TEST-2020

Binomial Theorem

1. If the coefficient of x^6 in $\left(x^3 + \frac{k}{x}\right)^6$ is 160, then k =

(a) 3 (b) 4
(c) 2 (d) none of these

2. The term independent of x in $\left(x^3 - \frac{1}{x^2}\right)^5$ is

(a) –10 (b) 10
(c) –20 (d) 20

3. If in the expansion of $(1 + x)^m (1 - x)^n$ the coeff. of x and x^2 are 3 and –6 respectively, then m is

(a) 6 (b) 9
(c) 12 (d) 24

4. The coefficient of x^{33} in the expansion of $\sum_{r=0}^{50} {}^{50}C_r (x-4)^{50-r} 3^r$ is

(a) ${}^{50}C_{33}$ (b) ${}^{50}C_{33}$
(c) $-{}^{50}C_{17}$ (d) ${}^{50}C_{17}$

5. If the coefficient of $(r + 1)$the term in the expansion of $(1 + x)^{2n}$ be equal to that of $(r + 3)$th term, then

(a) $n - r + 1 = 0$ (b) $n - r - 1 = 0$
(c) $n + r + 1 = 0$ (d) none of these

6. The middle term in the expansion of $\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^{10}$ is

(a) ${}^{10}C_5 x^{5/2}$ (b) ${}^{10}C_5 x^{-5/2}$
(c) ${}^{10}C_5$ (d) $-{}^{10}C_5$

7. The coefficient of x^4 in the expansion of $(1 + x + x^2 + x^3)^n$ is

(a) ${}^nC_4$ (b) ${}^nC_4 + {}^nC_2$
(c) ${}^nC_4 + {}^nC_2 + {}^nC_4.{}^nC_2$ (d) ${}^nC_4 + {}^nC_2 + {}^nC_1.{}^nC_2$

8. The coefficient of x^4 in the expansion of $(1 + x + x^2 + x^3)^n$ is

(a) ${}^{n+3}C_4 - {}^nC_1$ (b) ${}^nC_4$
(c) ${}^nC_2 + {}^nC_4$ (d) ${}^nC_2 + {}^nC_4 + {}^nC_4.{}^nC_2$

9. The number of non-zero terms in the expansion of $(2 + \sqrt{2}x)^7 + (2 - \sqrt{2}x)^7$ is

(a) 3 (b) 4

(c) 0 (d) 8

10. Greatest term in the expansion of $(2 + 3x)^{12}$ when $x = 5/6$ is

(a) $^{12}C_7.5^5.4$ (b) $^{12}C_8\frac{5^8}{2^4}$

(c) $^{12}C_7\left(\frac{5}{2}\right)^7.2^5$ (d) none of these

11. If the sum of odd numbered terms and sum of the even numbered terms in the expansion of $(x + a)^n$ are p and q respectively, then the value of $(x^2 - a^2)^n$ is

(a) $p^2 - q^2$ (b) $p^2 + q^2$

(c) 4pq (d) none of these

12. If the coefficients of x^6 and x^5 is the expansion of $\left(3+\frac{x}{4}\right)^n$ are equal then n =

(a) 17 (b) 47

(c) 77 (d) 67

13. If $(1 + x - 2x^2)^6 = 1 + a_1x + a_2x^2 + \ldots.. + a_{12}x^{12}$, then the expression $a_1 + a_3 + a_5 + \ldots. a_{11}$ is equal to

(a) 32 (b) 64

(c) –32 (d) 63

14. The sum of the coefficient in the expansion of $(\alpha^2x^2 - 2\alpha x + 1)^{51}$ vanishes, then α is equal to

(a) 2 (b) –1

(c) 1 (d) –2

15. If $x + y = 1$, then $\sum_{r=0}^{n} r^2.{}^nC_r x^r y^{n-r}$ equals

(a) nxy (b) $nx(x + yn)$

(c) $nx(nx + y)$ (d) none of these

16. If $(1 + x)^n = C_0 + C_1x + C_2x^2 + \ldots C_nx^n$, then $C_0^2 + C_1^2 + C_2^2 + \ldots + C_n^2$ is equal to

(a) $^{2n}C_{n-1}$ (b) $^{2n}C_n$

(c) $^{2n}C_{n+1}$ (d) none of these

17. The value of $C_0 + 3C_1 + 5C_2 + 7C_3 + \ldots + (2n + 1)C_n$ is equal to

(a) 2^n (b) $2^n + n.2^{n-1}$

(c) $2^n(n + 1)$ (d) none of these

18. The greatest coefficient in the expansion of $(1 + x)^{10}$ is

(a) 10 !/5 !.6! (b) 10 !/(5 !)^2

(c) 10 !/5 !.7 ! (d) none of these

19. The sum $\sum_{i=0}^{m}\binom{10}{i}\binom{20}{m-i}$, (where $\binom{p}{q}$ = 0 if $p < q$) is maximum when m is

(a) 5 (b) 10
(c) 15 (d) 20

20. In the expansion of $(1 + x)^{50}$, the sum of the coefficients of odd powers of x is

(a) 0 (b) 2^{49}
(c) 2^{50} (d) 2^{51}

21. $(1 - x)^{3/2}$ can be expanded in ascending powers of x if,

(a) $-1 < x < 1$ (b) $x < -1$
(c) $x > 1$ (d) none of these

22. The expansion of $\frac{1}{(4-3x^2)^{1/3}}$ is valid for

(a) $-\frac{2}{\sqrt{3}} < x < \frac{2}{\sqrt{3}}$ (b) $0 < x < 3$
(c) $-3 < x < 0$ (d) $0 < x < \infty$

23. The first negative term in the expansion of $(1 + x)^{7/2}$ is

(a) 3rd (b) 4th
(c) 5th (d) 6th

24. If $(1 - x)^{-n} = a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots$ then the value of $a_0 + a_1 + a_2 + \ldots + a_n$ is

(a) $\frac{2n!}{(n!)^2}$ (b) $\frac{n!}{(2n!)^2}$
(c) $\frac{2(2n)!}{(n!)^2}$ (d) $\frac{2n!}{(2n!)^2}$

25. The coefficient of x^{10} in the expansion of $\frac{(1+3x^2)}{(1-x^2)^3}$ is

(a) 33 (b) 66
(c) 99 (d) 132

26. The coefficient of x^6 in $(1 + x + x^2)^{-3}$ is

(a) 1 (b) 2
(c) 3 (d) 4

27. The coefficient of x^4 in the expression $(1 + 2x + 3x^2 + 4x^3 + \ldots)^{1/2}$ is

(a) 0 (b) 1
(c) 2 (d) 4

28. The approximate value of $(7.995)^{1/3}$ correct to four decimal places is

(a) 1.9995 (b) 1.9996
(c) 1.99c (d) 1.9991

29. The sum of the series $1 + \frac{1}{3^2} + \frac{1.4}{1.2}\frac{1}{3^4} + \frac{1.4.7}{1.2.3}\cdot\frac{1}{3^6} + \ldots$ is

(a) $\sqrt{\frac{3}{2}}$ (b) $\left(\frac{3}{2}\right)^{1/3}$

(c) $\sqrt{\frac{1}{3}}$ (d) $\left(\frac{1}{3}\right)^{1/3}$

30. If $\frac{x^2+x}{(1-x)} = a_1x + a_2x^2 + \ldots \infty$, $|x| < 1$ then

(a) $a_1 + a_2 = 4$ (b) $a_1 - a_2 = 3$

(c) $a_p = a_q$ (d) none of these

31. The coefficient of x^n in the series $1 + (a + bx) + \frac{(a+bx)^2}{2!} + \frac{(a+bx)^3}{3!} + \ldots \infty$,is

(a) $\frac{b^n}{n!}$ (b) $e^a.\frac{b^n}{n!}$

(c) $e^{-a}\frac{b^n}{n!}$ (d) none

32. The sum of the following infinite series is

$1 + \frac{2^3}{2!} + \frac{3^3}{3!} + \frac{4^3}{4!} + \ldots \infty$

(a) $2e$ (b) $3e$

(c) $4e$ (d) $5e$

33. The sum of the series $1 + \frac{1^2+2^2}{2!} + \frac{1^2+2^2+3^2}{3!} + \frac{1^2+2^2+3^2+4^2}{4!} + \ldots$ is

(a) $3e$ (b) $\frac{17}{6}e$

(c) $\frac{13}{6}e$ (d) $\frac{19}{6}e$

34. The sum of the series $\frac{1^2.2^2}{1!} + \frac{2^2.3^2}{2!} + \frac{3^2.4^2}{3!} + \ldots$ is

(a) $27\,e$ (b) $24\,e$

(c) $28\,e$ (d) $30\,e$

35. If $e^x = y + \sqrt{1+y^2}$, then the value of y is

(a) $(e^x - e^{-x})$ (b) $\frac{1}{2}(e^x - e^{-x})$

(c) $(e^x + e^{-x})$ (d) $\frac{1}{2}(e^x + e^{-x})$

36. If $a_n = \frac{\sum n}{n!}$, then the sum of the infinite series $\sum_{n=1}^{\infty} a_n$ is

(a) e (b) e^{-1}

(c) $\frac{3e}{2}$ (d) $\frac{e}{2}$

37. $\dfrac{\frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}+\ldots}{1+\frac{1}{3!}+\frac{1}{5!}+\ldots}$ equals

(a) $(e + 1)$ (b) $\frac{e-1}{e+1}$

(c) $(e - 1)$ (d) none

38. The sum of the series $\frac{1}{1.2} + \frac{1.3}{1.2.3.4} + \frac{1.3.5}{1.2.3.4.5.6} + \ldots \infty$ is

(a) e^{-1} (b) $e^{1/2} - 1$

(c) $e^{1/2} + e$ (d) none of these

39. If S = $\sum_{n=2}^{\infty} \frac{{}^nC_2}{(n+1)!}$; then S equals to

(a) $(e - 2)$ (b) $(e + 2)$

(c) $2e$ (d) $\left(\frac{e}{2}-1\right)$

40. If a = $\sum_{n=0}^{\infty} \frac{x^{3n}}{(3n)!}$, $b = \sum_{n=1}^{\infty} \frac{x^{3n-2}}{(3n-2)!}$

and c = $\sum_{n=1}^{\infty} \frac{x^{3n-1}}{(3n-1)!}$, then the value of $(a^3 + b^3 + c^3 - 3abc)$ is

(a) 1 (b) 0

(c) –1 (d) –2

41. The value of $\dfrac{(a-1)-\frac{1}{2}(a-1)^2+\frac{1}{3}(a-1)^3-..\infty}{(b-1)-\frac{1}{2}(b-1)^2+\frac{1}{3}(b-1)^3-..\infty}$ is

(a) $\log_a b$ (b) $\log_b a$

(c) $\log_b a^b$ (d) $\log_a b^b$

42. If $x^2y = (2x - y)$ and $|x| < 1$ and $A = y + \frac{y^3}{3} + \frac{y^5}{5} + \ldots \infty$

$B = 2\left(x+\frac{x^3}{3}+\frac{x^5}{5}+..\infty\right)$, then

(a) A > B (b) B > A

(c) A = B (d) $A^2 = B^2$

43. If α, β are the roots of the equation $ax^2 + bx + c = 0$, then

$\log a + (\alpha + \beta)x - \left(\frac{\alpha^2+\beta^2}{2}\right)x^2 + \left(\frac{\alpha^3+\beta^3}{3}\right)x^3 - \ldots$ is

(a) $\log(a + bx + cx^2)$ (b) $\log(a + bx - cx^2)$

(c) $\log(a - bx + cx^2)$ (d) none of these

44. The value of $\frac{1}{(2n+1)}+\frac{1}{3(2n+1)^3}+\frac{1}{5(3n+1)^5}+\ldots$ is

(a) $\log\left(1+\frac{1}{n}\right)$
(b) $\frac{1}{2}\log\left(1+\frac{1}{n}\right)$
(c) $2\log\left(1+\frac{1}{n}\right)$
(d) $\frac{1}{4}\log\left(1+\frac{1}{n}\right)$

45. The value of $\left(\frac{a-b}{a}\right)+\frac{1}{2}\left(\frac{a-b}{a}\right)^2+\frac{1}{3}\left(\frac{a-b}{a}\right)^3+\ldots\infty$ is

(a) $\log_e\frac{a}{b}$
(b) $\log_e\left(\frac{b}{a}\right)$
(c) $\log_e a+\log_e b$
(d) none of these

46. If A = $\frac{1}{(n+1)}+\frac{1}{2(n+1)^2}+\frac{1}{3(n+1)^3}+\ldots\infty$ and B = $\frac{1}{n}-\frac{1}{2n^2}+\frac{1}{3n^3}-\frac{1}{4n^4}+\ldots\infty$, then

(a) A + B = 0
(b) A – B = 0
(c) $A^2 + B^2 = 0$
(d) A < B

47. The value of $\frac{1}{1.2}+\frac{1}{3.4}+\frac{1}{5.6}+\ldots\infty$, is

(a) $4\log_e 2$
(b) $3\log_e 2$
(c) $2\log_e 2$
(d) $\log_e 2$

48. The sum of the infinite series $\frac{1}{23}+\frac{1}{45}+\frac{1}{67}+\ldots\infty$, is

(a) $1-\log_e 2$
(b) $1+\log_e 2$
(c) $1-2\log_e 2$
(d) $1+2\log_e 2$

49. If x, y, z are three consecutive positive integers, then

$\frac{1}{2}\log_e x+\frac{1}{2}\log_e z+\frac{1}{(2xz+1)}+\frac{1}{3}\left(\frac{1}{2xz+1}\right)^3+\ldots$ is equal to

(a) $\log_e x$
(b) $\log_e y$
(c) $\log_e z$
(d) $\log_e xy$

50. If S = $\frac{y-1-\frac{1}{2}(y-1)^2+\frac{1}{3}(y-1)^3-\ldots}{a-1-\frac{1}{2}(a-1)^2+\frac{1}{3}(a-1)^3-\ldots}$, then S is equal to

(a) $\log_e y$
(b) $\log_a y$
(c) $\log_e a$
(d) $\log_y a$